AN ANALYSIS OF YEAR 12 STUDENTS' PERFORMANCES ON BASIC ALGEBRA QUESTIONS

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This paper reports details concerning the performance of capable Year 12 Mathematics students in their final public examination in NSW. The focus of this analysis is on basic algebraic skills tested in the first question of the 2 unit paper. This topic has been encountered by students for a number of years and provides, potentially, an upper bound on what might be expected from such students in an examination. Success rates and the types of strategies employed by students are documented and discussed.

INTRODUCTION

The Higher School Certificate (HSC) is a critical examination for nearly 60,000 students in NSW. In Mathematics, there are five courses of study available for students to choose, and, surprisingly, when compared with other states in Australia, nearly 95% of students opt to take one of these courses. The courses are designed to cater for students of different abilities and aspirations. Of interest to this research are the performances, on a common paper, of students who choose the courses referred to as 2 unit related and 3 unit Mathematics. These two courses are designed for students who wish to take programs of study at university which require tertiary mathematics (such as economics or biology) and those who wish to pursue a program in tertiary mathematics, respectively. As such, students in this sample represent the top 50% – 95% of the student cohort in terms of overall achievement in Mathematics. (Another course referred to as 4 unit Mathematics is undertaken by the very best students in Mathematics who represent the top 5% of the student cohort in terms of mathematical ability. This group does not sit for the common 2 unit paper and is not included in the analysis below.)

Similiar to other large public examination systems, student results are reported using norm-referenced measures. These provide an indication to the public about the relative standing of students in terms of their relationship with their peers, and a mark that reflects, in a general way, students' achievements. However, there is no indication in these marks of whether the cohort has performed better or worse than previous years nor what the mark actually means. As an initial attempt to address these two issues this paper reports upon a small aspect of a large study which is designed to investigate actual student achievement in NSW Higher School Certificate examinations in Mathematics. Other reports on this investigation have been presented in other forums (Pegg, 1997; Pegg, Hadfield & Hastings, 1997). This form of information has not been available previously and represents a first attempt in NSW at identifying what students, at the end of their secondary schooling, know, understand and can do.

BACKGROUND

Student performances in Mathematics at the HSC are reported by a standardised mark usually set at a mean of 60 and standard deviation of 12.5. This, together with a percentile banding, provides an indication of a student's performance. The scaling procedure is such that the mark reported is one which the general public can interpret. This means that marks of 80 or above represent high achievement whereas marks below 50 are usually considered to represent poor achievement or failure.

However, while the marks allow a ranking of students in the State, they do not (and are not designed to) indicate a measure of what students know and understand. As a balance to this position there are plans emerging (not unique to NSW) for the reporting of the examination results to take on a more qualitative stance. However, before this can be achieved it is important that analyses are undertaken to provide some indication of the competencies shown currently by students to provide the basis for realistic criteria to be established. This process mirrors current developments in business and related fields associated with benchmarking (Bartos, 1994; Zairi, 1992).

This paper reports on an investigation into this area by documenting the performances of the selected cohort of students, over three years, on basic algebraic questions which would be seen to be among the most straight forward, familiar and less abstract of the Mathematics items tested.

DESIGN

With the support of the Board of Studies of NSW, 1000 scripts were randomly selected for each of the ten questions asked in the HSC 2 unit related examination papers for 1995, 1996 and 1997. All indications of student numbers and centres of examination were removed from the scripts. It is this sample upon which the data reported in this paper are based. Initially a small subset of papers (n=100) were categorised. A co-worker familiar with the Mathematics content was provided with the scripts and the coding categories. The agreement of categoriastion was 94%. When this was completed the full sample of scripts was analysed.

The questions chosen for the analysis are provided below. Because of space restrictions only student performances on five questions are documented here.

From 1995

Q1 (a) Factorise $9x^2 - 16$.	(1 mark)
Q1 (d) Simplify $x/3 + (3x - 1)/2$	(2 marks)

From 1996

Q1 (b) Factorise $2x^2 + 3x - 2$.	(2 marks)
Q1 (d) Simplify $2/3 - (x - 1)/4$	(2 marks)

From 1997

Q1 (b) Simplify $(2 - 3x) - (5 - 4x)$.	(2 marks)

To facilitate the discussion, the analysis is considered under two broad sections.

RESULTS

Section 1

This section considers the two question parts Q1(a) (1995), Q1(b) (1996). In both cases students were expected to factorise a quadratic expression. A summary of the results is provided in Table 1.

Table 1
Percentage of Students Attaining Allocated Marks

	No attempt	0	1	2
Q1(a) (1995)	4	23	73	N/A
Q1(b) (1996)	4	17	6	73

For Q1(a) (1995), of the 73% who were awarded 1 mark (i.e., those who were correct), 3% re-interpreted the question and formed an equation (i.e., equated the expression provided with zero) and solved this to obtain $x = \pm 4/3$.

Of the 23% who were awarded zero marks, five approaches were identified. In these cases students:

1. confused the expression given with a binomial squared, i.e., $(3x - 4)^2$ 4%
2. did not factorise as a difference of two squares, e.g.,
   - $9(x - 4)(x + 4)$
   - $(x - 3)(x + 4)$
   - $(9x + 4)(x - 4)$ 6%
3. attempted to solve $9x^2 - 16 = 0$, e.g.,
   - $9x^2 = 16$
     $x = \sqrt{7}$ 5%
4. did not write answer in a factorised form, e.g.,
   - $3x^2 - 4$
   - $(x - 3)(x + 3) - 16$
   - $12x$ 5%
5. found a common factor, e.g.,
   - $9x(x - {}^{16}/_{9x})$
   - $3(3x^2 - 5\,{}^{1}/_{3})$
   - $2({}^{9x^2}/_{2} - 8)$ 3%

Overall, the majority of the 23% of students who were incorrect on this question part did not appear to make errors in factorising the difference of two squares. Instead, students did not seem to recognise the expression as a difference of two squares.

For Q1(b) (1996), 73% of students were correct (were awarded 2 marks). Of these, approximately 10% of students also found a solution to $2x^2 + 3x - 2 = 0$. The most common method indicated by those who were correct was the 'cross-method' (45% of students), however, 20% of students who provided the correct answer showed no working.

In all, 23% of students were incorrect. About half of these attempted to factorise the expression, but made errors. The majority of students who provided an incorrect response used a method other than the 'cross-method'.

Of those students who were incorrect, 40%, (i.e., 9% of total), did not factorise at all. Approximately half of the students attempted to solve $2x^2 + 3x - 2 = 0$ but were unable to produce the correct answer. Others attempted to simplify the expression, however, these students demonstrated very poor algebra skills. Interestingly, one student from the sample registered five incorrect attempts to find two binomials whose product was $2x^2 + 3x - 2$. There were five incorrect strategies identified. In these cases students:

1. provided incorrect factors, such as: $(2x + 1)(x - 2)$ 12%
2. provided incorrect factors and solutions to $2x^2 + 3x - 2 = 0$ 2%
3. applied the quadratic formula to solve $2x^2 + 3x - 2 = 0$ 4%
4. applied inappropriate algebraic simplification techniques 4%
5. applied trial and error techniques by selecting two binomial products and multyplying them out, e.g., $(x + 3)(2x - 2)$ 1%

In this question, apart from the 4% who did not attempt the question, all but 8% of the students produced two binomial products for their answer. Also of interest were the attempts by 13% of the students who changed what was asked and set up and solved an equation.

These two questions, asked of different students in different years show remarkably similiar patterns of results. In both cases 73% were correct and approximately the same number of students chose to solve an equation.

Section 2.

This section considers the three question parts Q1(d) (1995), Q1(d) (1996), and Q1(b) (1997). Q1(d) (1995) and (1996) involved the simplification of algebraic fractions, whereas Q1(b) (1997) required removing brackets and collecting like terms. A summary of the results is provided in Table 2.

Table 2
Percentage of Students Attaining Allocated Marks

	No attempt	0	1	2
Q1(a) (1995)	3	17	15	65
Q1(b) (1996)	5	11	34	50
Q1(b) (1997)	0	15	10	75

For Q1(d) (1995), 65% of the students were correct and a further 15% were able to make some progress towards the solution. Five categories of incorrect responses were identified. In these cases, students:

1. manipulated correctly the fractions, but did not simplfy the resulting expression — 5%
2. made a calculation error, e.g., — 5%
3. experienced problems with adding fractions
   - denominator omitted — 14%
   - faulty knowledge of addition of fractions demonstrated, e.g.,
     $x/3 + (3x - 1)/2 = (4x - 1)/5$
     $x/3 + (3x - 1)/2 = 3/x \times (3x - 1)/2$ — 5%
4. made a cancelling error simplifying the answer, e.g.,
   $(11x - 3)/6 = (11x - 1)/2$ — 1%
5. attempted to solve an equation by making the expression equal to zero — 2%

Overall, 32% of students were incorrect. About half of these students showed very little knowledge of adding fractions, especially algebraic fractions, e.g., many had no common denominator in their answers, or added the given numerators.

Q1(d) (1996) was similiar to the previous question, discussed above, except that this question required the subtraction of two algebraic expressions and not the addition. As expected the number of students correct was much lower here than for the previous question, i.e., 50% compared with 65%. There were five categories of incorrect responses identified. In these cases students:

1. manipulated correctly the fractions, but did not simplfy the resulting expression — 4%

2. made a calculation error, e.g.,
   $(8 - 3x + 3)/12 = (12 - 3x)/12$ 9%
3. ignored the effect of the minus sign on the second term, e.g.,
   $8/12 - (3x - 3)/12 = (8 - 3x + 3)/12 = (5 - 3x)/12$ 12%
4. experienced problems with subtracting fractions
   • denominator omitted, e.g.,
   $8/12 - (3x - 3)/12 = 8 - 3x + 3 = 11 - 3x$ 6%
   • faulty knowledge of subtraction of fractions demonstrated, e.g.,
   $8/12 - (3x - 3)/12 = 8(3x - 3)/12$ 12%
5. attempted to solve an equation by making the expression equal to zero 2%

The large difference in performance on this question compared with the previous question is now able to be explained. The presence of the minus sign meant that 12% of the students made a specific error (see strategy 3 above). When this is taken into account the performance of students on this question across both years is remarkably similiar.

For Q1(b) (1997), 75% of students were correct. This is the best result of all the items reported in this paper.

About one third of those incorrect (i.e., 9% of total) simplified (correctly or incorrectly) then solved for x. Others, 40% of incorrect students (10% of total), calculated the expression as a product, i.e., $(2 - 3x)(-5 + 4x)$.

Most of the remaining errors occurred because students did not change the sign when removing the brackets. Considering that this aspect of simplifying is emphasised in class from the time students do this topic in Year 9, it is surprising that approximately 10% of students still make this error.

In all there were four categories of errors identified. Students:

1. simplified correctly and solved for x 7%
2. simplified incorrectly and solved for x 2%
3. calculated the binomial product $(2 - 3x)(-5 + 4x)$ 10%
4. made errors in collecting like terms, e.g.,
   $(2 - 3x) - (5 - 4x) = 2 - 3x - 5 - 4x = -3 - 7x$ 6%

Overall, 82% of students were able to successfully simplify the expression. However, 7% of students went further and equated the expression to zero and solved an equation. Given that this form of question would have been encountered in Year 8, and students would have completed many examples of a similiar nature over the next four years this result is disappointing.

DISCUSSION

In summary, these data suggest that approximately:

- three quarters of the cohort can successfully factorise a binomial or trinomial quadratic;
- two thirds of the cohort can add two algebraic fractions but when a subtraction is introduced it drops the success rate down to about 50%; and
- three quarters of the cohort can successfully expand out brackets involving a minus sign and collect like terms.

Significant problems identified in student responses include:

- an inability to recognise the difference of two squares;
- a tendency to want to reinterpret questions so that they became equations to be solved;
- an inability to understand the place of numerators and denominators in fractions; and
- difficulties in monitoring basic algebraic and arithmetic operations while undertaking the solution process.

The cohort of students in this sample is drawn from the top 50% of the Year 12 ability range in Mathematics in NSW. It is from this group of students that future mathematics teachers, engineers, and the like are predominantly drawn. As such this group represents an important, even significant, body of students to a society in terms of future responsibilities. Significantly for many of these students, competence at Mathematics is going to be an important pre-requisite.

The questions reported are among the most straight forward and least abstract of all the questions posed in the two courses. However, with overall success rates of between 75% and 50% it is clear that this perception of low difficulty is not a view that students would seem to share. A confounding variable is that the ideas tested and the very question types are ones that all students would have encountered on many occasions over the previous four years. Why, after all this, the students' performances are as low as they are is difficult to explain.

When these tasks are considered from a cognitive perspective the results are also disconcerting. One would expect the working memory demands of such questions to be relatively light. Because of their preponderance in other years and overall familiarity, one would expect that all students would have developed routines and procedures for attempting such questions. In addition, the 'traps' or typical mistakes made in questions with minus signs must have been encountered many hundreds of times and should come as no surprise to students of this calibre. However, the results show that for many of these students these assumptions are not true and that there responses are no better than would be expected of many Year 9 students. It is as if these students have not progressed despite four years of continuing exposure to attempting such questions.

CONCLUSION AND IMPLICATIONS

The results summarised in this paper are taken from students' attempts at undertaking an external examination. Because of this, it is difficult to pass off the results in terms of students not wanting or choosing to give their best. Instead, it could be argued that given the importance of the examination and the preparation time allocated (in terms of trial papers and extensive revision over several weeks) that the responses provided by students should represent an optimum performance. This being the case, then there is little opportunity for complacency in the findings for Mathematics educators, parents or students.

Given that these questions were selected because of their familiarity and ease, it will be interesting to identify the trends in other topics tested in which the material is more abstract and the time exposure to the ideas is considerably less than was the case with the questions surveyed here. It is only by such analysis that a clear picture can be developed of the true state of Mathematics learning in our senior years. However, the process of collecting these data is time consuming and expensive.

This research does highlight the need to go beyond reporting student performance at the HSC in terms of marks which allow norm-based referencing. It is also clear that procedures which are currently in place actually mask the real extent or weaknesses of student understandings. While it is not a panacea, it does seem, from our perspective, that if teachers were made aware of their students' performance in terms of what they actually know and understand, then a serious commencement could be made in helping students reach their true potential.

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